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Exam I Review, MTH 221 , Fall 2010

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QUESTION 1. Let $A \begin{bmatrix} 2 & 2 & -1 & 4 \\ -2 & 3 & 0 & 1 \\ 1 & 2 & -2 & 4 \end{bmatrix}$ and $K = \begin{bmatrix} 1 & 0 & 2 \\ -4 & 2 & 2 \\ 0 & 1 & 3 \\ -2 & 1 & 1 \end{bmatrix}$

(i) Find the 2nd column of AK

(ii) Find the third row of KA

(iii) Find the (3, 4)-entry KA

(iv) Find the trace of AK

(v) Solve the system $AX = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$

QUESTION 2. Let $A = \begin{bmatrix} 3 & 2 \\ -4 & 6 \end{bmatrix}$. Write A as a linear combination of a symmetric and a skew symmetric matrix. (you must Find H (symmetric), W (skew symmetric) and two constants j, i such that $A = jH + iW$)

QUESTION 3. Let $H = \begin{bmatrix} 3 & 2 & b \\ -3 & -2 & 5 \\ 6 & c & 10 \end{bmatrix}$

(i) For what values of b, c does the system $HX = \begin{bmatrix} 5 \\ -5 \\ 7 \end{bmatrix}$ have a unique solution?

(ii) For what values of b, c does the system in (i) have infinitely many solutions?

(iii) For what values of b, c is the system inconsistent?

(iv) For what values of b, c will H be nonsingular(invertible)?

QUESTION 4. Use row operations only in order to calculate $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -0.5 \end{bmatrix} \begin{bmatrix} 6 & 12 \\ -4 & 10 \end{bmatrix}$

QUESTION 5. Let A be a 4×4 matrix. Given

$$A \underbrace{3R_1 + R4 \rightarrow R_4} A_1 \underbrace{R_3 \leftrightarrow R_2} A_2 \underbrace{-3R_1} A_3 \underbrace{-4R_1 + R_2 \rightarrow R_2} A_4 = \begin{bmatrix} 1 & 2 & 2 & -4 \\ 0 & 0 & 3 & -2 \\ -1 & 4 & 2 & 2 \\ -1 & -2 & -2 & 8 \end{bmatrix}$$

(i) Find $det(A)$.

(ii) Find $det(A_3)$

(iii) Find a matrix B such that $BA = A_4$

(iv) Find a matrix C such that $CA = A_3$

(v) Find $det(2A_4A_2)$

(vi) Is A nonsingular? if yes find $det(0.5A^{-1}A_1)$.

(vii) Find $det(0.2(A_3A_4)^T)$

(viii) Find elementary matrices E_1, E_2, E_3 such that $E_1E_2E_3A = A_3$.

(ix) Find A_4^{-1}

(x) Find the $(2,4)$-entry of A_3^{-1}

QUESTION 6. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ Given $det(A) = 21.23$ Consider the following system $AX = \begin{bmatrix} 3.2a_2 \\ 3.2a_5 \\ 3.2a_8 \end{bmatrix}$. Solve for x_1, x_2, and x_3.

QUESTION 7. (a)Find a 3×4 matrix A such that $\begin{bmatrix} 3 & 2 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix} A + \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} = 2A + \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

(b) Find a 2×2 matrix such that $A \begin{bmatrix} 2 & 4 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & 4 \end{bmatrix}$

QUESTION 8. Use the adjoint method to find the inverse of $A = \begin{bmatrix} 3 & 2 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

QUESTION 9. Given A is a 3×3 matrix such that $A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & -3 & 3 \end{bmatrix}$. Find the solution for the system $AX = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

QUESTION 10. (a) Find det(A) where $A = \begin{bmatrix} 3 & -2 & 2 \\ 6 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix}$

(b) Find det(A) where $A = \begin{bmatrix} 1 & 2 & 2 & -4 \\ -1 & -2 & 3 & -2 \\ -1 & 4 & 2 & 2 \\ 4 & 8 & 8 & -15 \end{bmatrix}$

QUESTION 11. Find the LU-Factorization of $A = \begin{bmatrix} 1 & 2 & 2 & -4 \\ -1 & 4 & 2 & 2 \\ -1 & -2 & 3 & -2 \\ 4 & 8 & 8 & -15 \end{bmatrix}$

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